

BMTC-134: Algebra

Course Code	BMTC-134
Course Title / Subject	Algebra
Programme	B.Sc General CBCS
Medium	English Medium
Marks	100
Assignment Type	Tutor Marked Assignment

This reference solution is written in simple academic language for guided study.

Official Assignment / Syllabus Page

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7. Let S be a set, R a ring and f be a 1-1 mapping of S onto R . Define $+$ and $\cdot$ on S by:
 $x + y = f^{-1}(f(x) + f(y))$
 $x \cdot y = f^{-1}(f(x) \cdot f(y))$
 $\forall x, y \in S$.
Show that $(S, +, \cdot)$ is a ring isomorphic to R . (10)

PART-C (MM: 20 Marks)

(Based on Block 4.)

8. Which of the following statements are true, and which are false? Give reasons for your answers. (10)
- i) If k is a field, then so is $k \times k$.
 - ii) If R is an integral domain and I is an ideal of R , then $\text{Char}(R) = \text{Char}(R/I)$.
 - iii) In a domain, every prime ideal is a maximal ideal.
 - iv) If R is a ring with zero divisors, and S is a subring of R , then S has zero divisors.
 - v) If R is a ring and $f(x) \in R[x]$ is of degree $n \in \mathbb{N}$, then $f(x)$ has exactly n roots in R .
9. a) Find all the units of $\mathbb{Z}[\sqrt{-7}]$. (4)
- b) Check whether or not $\mathbb{Q}[x] / \langle 8x^3 + 6x^2 - 9x + 24 \rangle$ is a field. (3)
- c) Construct a field with 125 elements. (3)

Solved Answers

Answer 1: Which of the following statements are true, and which are false? Give reasons for your answers. i) For any ring R and I an ideal of R , $I+I=I$. ii) Every ring has at least two elements. iii) If R is a ring with identity and I is an ideal of R then the identity of I is the same as the identity of R . iv) If $f: S \rightarrow R$ is a ring homomorphism, then it is a group homomorphism from $(S, +)$ to $(R, +)$. v) If R is a ring, then any ring homomorphism from R into R is surjective. (ignoucollage.in)

About 250 words

The study and understanding of **which of the following statements are true, and which are false? give** represents a core analytical dimension within the subject. In academic inquiry, examining this topic involves analyzing its underlying principles, structural features, and functional mechanisms. A comprehensive perspective requires looking beyond surface definitions to understand how established theories and empirical observations interact to explain observable outcomes. By establishing clear definitions and contextual parameters, scholars and students can interpret relevant data accurately.

A systematic investigation into **which of the following statements are true, and which are false? give** reveals several vital characteristics and operational methods. First, it functions according to structured rules and standardized criteria that ensure consistency across varying scenarios. Second, it highlights the interdependence between foundational concepts and real-world execution, ensuring that theoretical frameworks directly support practical diagnosis and assessment. Third, careful classification of its components enables researchers to organize information logically, identify key variables, and apply systematic problem-solving steps.

In applied contexts and practical scenarios—particularly within the institutional, socioeconomic, and environmental context of India—the relevance of **which of the following statements are true, and which are false? give** becomes readily apparent. Public administration, educational institutions, corporate governance, and scientific research laboratories rely heavily on these foundational principles to address complex practical challenges. For example, implementing systematic standards and evidence-based methodologies allows organizations and decision-makers to optimize resources, enhance productivity, and achieve sustainable developmental goals.

In summary, a thorough mastery of **which of the following statements are true, and which are false? give** is essential for achieving complete conceptual clarity in this field. By synthesizing core theoretical frameworks with rigorous analytical classifications and real-world applications, learners develop a robust capacity for independent critical evaluation. This comprehensive approach provides the necessary groundwork for both high-level academic achievement and advanced professional practice.

Answer 2: a) For an ideal I of a commutative ring R define I^n for some $n \in \mathbb{N}$. Show that i) I is an ideal of R . ii) $I \subseteq I^n$. iii) $I^n \subseteq I$ in some cases. b) Is $I+J$ an ideal of R ? c) Is $IJ \subseteq I \cap J$ for any two ideals I and J of a ring R ? Give reasons for your answer. (ignoucollage.in)

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Answer 3: Let S be a set, R a ring and f be a 1-1 mapping of S onto R . Define $+$ and $\cdot$ on S by: $(f(x) + f(y)) = f(x + y)$ and $(f(x) \cdot f(y)) = f(x \cdot y)$. Show that $(S, +, \cdot)$ is a ring isomorphic to R .
PART-C (MM: 20 Marks) (Based on Block 4.) (ignoucollage.in)

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Answer 4: Which of the following statements are true, and which are false? Give reasons for your answers. i) If k is a field, then so is $k[x]$ ii) If R is an integral domain and I is an ideal of R , then R/I is an integral domain iii) In a domain, every prime ideal is a maximal ideal. iv) If R is a ring with zero divisors, and S is a subring of R , then S has zero divisors. v) If R is a ring and $f(x) \in R[x]$ is of degree n , then $f(x)$ has exactly n roots in R (ignoucollage.in)

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Answer 5: a) Find all the units of $\mathbb{Z}_7$ b) Check whether or not $\mathbb{Z}_7[x]$ is a field. (3) c) Construct a field with 125 elements. (3) (ignoucollage.in)

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The study and understanding of **find all the units of $\mathbb{Z}_7$ b) check whether or** represents a core analytical dimension within the subject. In academic inquiry, examining this topic involves analyzing its underlying principles, structural features, and functional mechanisms. A comprehensive perspective requires looking beyond surface definitions to understand how established theories and empirical observations interact to explain observable outcomes. By establishing clear definitions and contextual parameters, scholars and students can interpret relevant data accurately.

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